4.1 Anti-Derivatives and Family of Functions

Recall: In "differential" calculus basic problem was find instantaneous rate of change of a function. (ie. slope of tangent line for a curve)

$y = x^3 \longrightarrow y' = 3x^2$

In "Integral" calculus, determine the curve (function) if you are given the derivative. (slope of tangent) Integration

Thus: Differentiation and Integration are inverse operations. In integration you are given the derivative of a function and asked to find the original function.

Consider: Find the Anti-derivative of function $F(x) = \int 2x(dx)$

integral

Indefinite - integral

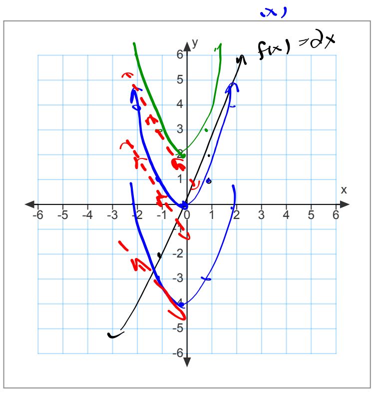
$F(x) = \int 2x\,(dx)$ ← VARIABLE

original anti-derivative

$f(x)$

$f(x) = 2x$

$F(x) = x^2 + C$



Indefinite Integrals and families of functions. Many functions have the same derivatives. (Note: All functions are related to each other using transformations (Vertical Shifts Up/Down)

Caution: To find a definite integral (exact function more information is required)

In general notation used for anti-derivative or indefinite integration:

$$y = \int f(x)dx = F(x) + C$$

Anti-derivative belongs to

family of functions

The following are your differentiation rules and their related integration rules.

Basic Integration Rules

Differentiation Formula		*Integration Formula*	
$\frac{d}{dx}[C] = 0$	$y=4 \quad y'=0$	$\int 0\,dx = C$	$\int(0)\,dx = C$
$\frac{d}{dx}[kx] = k$	$y=3x \quad y'=3$	$\int k\,dx = kx + C$	$\int -7(dx) = -7x + C$
$\frac{d}{dx}[kf(x)] = kf'(x)$		$\int kf(x)\,dx = k\int f(x)\,dx$	$\int 4x$
$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$		$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$	
$\frac{d}{dx}[x^n] = nx^{n-1}$	$y=x^3 \quad y'=3x^2$	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$	Power Rule
$\frac{d}{dx}[\sin x] = \cos x$		$\int \cos x\,dx = \sin x + C$	
$\frac{d}{dx}[\cos x] = -\sin x$		$\int \sin x\,dx = -\cos x + C$	
$\frac{d}{dx}[\tan x] = \sec^2 x$		$\int \sec^2 x\,dx = \tan x + C$	
$\frac{d}{dx}[\sec x] = \sec x \tan x$		$\int \sec x \tan x\,dx = \sec x + C$	

$\frac{d}{dx}[\sec x] = \sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$
$\frac{d}{dx}[\cot x] = -\csc^2 x$	$\int \csc^2 x\,dx = -\cot x + C$
$\frac{d}{dx}[\csc x] = -\csc x \cot x$	$\int \csc x \cot x\,dx = -\csc x + C$
$\frac{d}{dx}[e^x] = e^x$	$\int e^x\,dx = e^x + C$
$\frac{d}{dx}[a^x] = (\ln a)a^x$	$\int a^x\,dx = \left(\frac{1}{\ln a}\right)a^x + C$
$\frac{d}{dx}[\ln x] = \frac{1}{x}, x > 0$	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$

$n = -1$

Find anti-derivative (indefinite integral) for:

A) $\int dx$

$F(x) = \int (1)\, dx$

$F(x) = \int (1)\, dx + \int (0)\, dx$

$F(x) = x + C$

Anti deri

B) $\int (3x)dx$

$F(x) = 3\int x^1 (dx)$

$F(x) = 3\left(\frac{1}{2}x^2\right) + C$

$F(x) = \frac{3}{2}x^2 + C$

To check differentiate

C) $\int (x+2)dx$

f(x)

$F(x) = \int x^1 (dx) + \int 2(dx)$

$F(x) = \frac{1}{2}x^2 + 2x + C$

$F(x) = \frac{x^2}{2} + 2x + C$

or

$F(x) = \frac{x^2+4x}{2} + C$

D) $\int 3\sqrt{x}dx$

$F(x) = 3\int x^{\frac{1}{2}} (dx)$

$F(x) = 3\left(\frac{2}{3}x^{\frac{3}{2}}\right) + C$

$F(x) = 2x^{\frac{3}{2}} + C$

$F(x) = 2\sqrt{x^3} + C$

E) $\int \left(\frac{2}{x^7} - \frac{x^5}{2}\right)dx$

$F(x) = 2\int x^{-7} - \frac{1}{2}\int x^5\, dx$

$F(x) = 2\left(\frac{-1}{6}x^{-6}\right) - \frac{1}{2}\left(\frac{1}{6}x^6\right) + C$

$F(x) = \frac{-1}{3x^6} - \frac{x^6}{12} + C$

$F'(x) = \cancel{2x^{-7}} - \frac{1}{2}x^5$

F) $\int \left(\sqrt[3]{x}\right)dx$

$F(x) = \int x^{\frac{1}{3}} (dx)$ $+\frac{3}{3}$

$F(x) = \frac{3}{4}x^{\frac{4}{3}} + C$

$F(x) = \frac{3\sqrt[3]{x^4}}{4} + C$

G) $\int \frac{2x^2-7x}{x^4}dx$

$F(x) = \int \frac{2}{x^2} - \frac{7}{x^3}\, dx$

$F(x) = 2\int x^{-2+1} - 7\int x^{-3+1} (dx)$

$F(x) = 2\left(\frac{1}{-1}x^{-1}\right) - 7\left(\frac{-1}{2}x^{-2}\right)$

$F(x) = -2x^{-1} + \frac{7}{2}x^{-2} + C$

H) $\int (x+2)(x-3)dx$

sum of terms instead

$= \int x^2 - x - 6\ dx$

$F(x) = \int x^{2+1} - \int x^{1+1} - \int 6\ (dx)$

$\boxed{F(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x + C}$

I) $\int \frac{x+1}{\sqrt{x}}dx$

$= \int x^{\frac{1}{2}+1} + x^{-\frac{1}{2}+1}\ dx$

$= \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$

Indefinite Integrals

J) $\int \frac{1}{x}dx$

$= \int x^{-1}\ dx$

$\boxed{= \cancel{\frac{1}{0}x^0}}$? undefined

Recall $y = \ln x$

$y' = \frac{1}{x}$

$\boxed{= \ln x + C}$

Since no integration rule exists for prod rule, quotient rule, chain rule, must re-write all functions as sum of terms! (Power rule applies)